Student Workbook

Arithmetic Sequences

Name: ______________________ Date: ____________ Class: ______

ESSENTIAL QUESTION

How can we use a constant difference to describe, extend, and apply arithmetic sequences in real-world situations?

Lesson Overview

An arithmetic sequence is a list of numbers where each term is obtained by adding a fixed value — the common difference d — to the previous term. Arithmetic sequences model constant-rate change: salary raises, staircase tile counts, and evenly spaced measurements.

$$d = a_n - a_{n-1}$$
$$a_n = a_1 + (n-1)d$$
$$a_n = a_{n-1} + d$$

Common Difference	$d = a_n - a_{n-1}$ — the constant amount added to each term to get the next; constant for all consecutive pairs.
Explicit Formula	$a_n = a_1 + (n - 1)d$ — gives any term directly from its position.
Recursive Formula	$a_n = a_{n-1} + d$, with a_1 given — each term built from the one before it.
Arithmetic Means	Terms inserted evenly between two values so the whole list is arithmetic.

Worked Examples

EXAMPLE 1

Find the first 5 terms and the common difference for the sequence with $a_1 = 7$ and $d = -3$.

- Start with $a_1 = 7$.
- $a_2 = 7 + (-3) = 4$; $a_3 = 4 + (-3) = 1$; $a_4 = 1 + (-3) = -2$; $a_5 = -2 + (-3) = -5$

Answer: First 5 terms: 7, 4, 1, −2, −5; d = −3

EXAMPLE 2

Find a_{10} for the arithmetic sequence with $a_1 = 4$ and $d = 6$ using the explicit formula.

- Explicit formula: $a_n = a_1 + (n - 1)d$
- Substitute: $a_{10} = 4 + (10 - 1)(6) = 4 + 9·6 = 4 + 54$

Answer: $a_{10} = 58$

EXAMPLE 3

Given $a_3 = 11$ and $a_7 = 27$, find d and write the explicit formula.

- $d = (a_n - a_m)/(n - m)$ with n=7, m=3: $d = (27 - 11)/(7 - 3) = 16/4 = 4$
- Find a_1: $a_3 = a_1 + (3-1)(4) \rightarrow 11 = a_1 + 8 \rightarrow a_1 = 3$
- $a_n = 3 + (n - 1)\cdot 4 = 4n - 1$

Answer: $d = 4$; $a_n = 4n - 1$

EXAMPLE 4

Insert 3 arithmetic means between 5 and 25.

- With 3 means inserted, there are 5 terms total: $a_1 = 5$, $a_5 = 25$.
- $d = (25 - 5)/(5 - 1) = 20/4 = 5$
- $a_2 = 5+5 = 10$; $a_3 = 10+5 = 15$; $a_4 = 15+5 = 20$

Answer: The arithmetic means are 10, 15, and 20.

EXAMPLE 5

A job starts at $42,000 with $2,500 annual raises. Find the salary in year 8.

- Arithmetic with $a_1 = 42{,}000$ and $d = 2{,}500$. Use $a_n = a_1 + (n - 1)d$ with $n = 8$.
- $a_8 = 42{,}000 + (8 - 1)(2{,}500) = 42{,}000 + 17{,}500$

Answer: The salary in year 8 is $59,500.

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Find the first 5 terms of the arithmetic sequence with $a_1 = 2$ and $d = 5$.

Hint: Add d = 5 repeatedly starting from $a_1 = 2$. Use $a_n = a_{n-1} + 5$.

2 Find a_{12} for the arithmetic sequence with $a_1 = -3$ and $d = 4$.

Hint: Use $a_n = a_1 + (n - 1)d$. Substitute $n = 12$, $a_1 = -3$, $d = 4$.

3 Find d and the explicit formula given $a_2 = 9$ and $a_5 = 21$.

Hint: Use $d = (a_5 - a_2)/(5 - 2)$. Then find a_1 using $a_2 = a_1 + d$.

4 Insert 2 arithmetic means between 8 and 20.

Hint: With 2 means, there are 4 terms total: $a_1 = 8$, $a_4 = 20$. Find $d = (20 - 8)/(4 - 1)$.

5 A theater has row 1 with 20 seats, and each row adds 3 seats. How many seats are in row 15?

Hint: Arithmetic with $a_1 = 20$, $d = 3$. Use $a_n = a_1 + (n - 1)d$ with $n = 15$.

Key Vocabulary

Arithmetic sequence	A sequence where each term differs from the previous by a constant amount.
Common difference (d)	The constant value added to each term to get the next: $d = a_n - a_{n-1}$.
Explicit formula	A formula that gives the nth term directly: $a_n = a_1 + (n-1)d$.
Recursive formula	A formula that defines each term using the previous term: $a_n = a_{n-1} + d$.
Arithmetic means	Terms inserted between two values so that all terms form an arithmetic sequence.

Quick Check Quiz

Circle the letter of the correct answer for each question.

1 **What is the common difference of the sequence 2, 9, 16, 23, 30?** Multiple Choice

A 5
B 7
C 9
D 6

2 **Which formula correctly gives the nth term of an arithmetic sequence?** Multiple Choice

A $a_n = a_1 \cdot d^{n-1}$
B $a_n = a_1 + (n-1)d$
C $a_n = a_1 + nd$
D $a_n = n \cdot d + a_0$

3 **Find a_8 for the sequence with $a_1 = 5$ and $d = -4$.** Multiple Choice

A −23
B −27
C −22
D −19

4 **How many arithmetic means are inserted if the total sequence has 6 terms?** Multiple Choice

- **A** 3
- **B** 4
- **C** 5
- **D** 6

5 **Given $a_2 = 7$ and $a_6 = 23$, what is d?** Multiple Choice

- **A** 3
- **B** 4
- **C** 5
- **D** 6

Common Mistakes to Avoid

✗ Using n instead of (n − 1) in the explicit formula: $a_n = a_1 + n \cdot d$.

✓ The correct formula is $a_n = a_1 + (n - 1)d$ because the first term requires 0 additions of d.

✗ Assuming a sequence is arithmetic after checking only one pair of consecutive terms.

✓ Check that the difference is constant for ALL consecutive pairs before concluding it is arithmetic.

✗ When inserting k arithmetic means, using k+1 as the number of intervals.

✓ Inserting k means creates k+2 total terms and k+1 equal intervals, so d = (last − first)/(k+1).

✗ Confusing the term number n with the term value a_n.

✓ n is the position (1, 2, 3, ...); a_n is the value at that position.

Math Tips

★ To verify a sequence is arithmetic, subtract consecutive terms — the differences must all be equal.

★ The explicit formula $a_n = a_1 + (n-1)d$ is equivalent to slope-intercept form $y = mx + b$, where d is the slope.

★ A negative common difference means the sequence is decreasing.

★ When finding a_1 from a later term, work backwards: $a_1 = a_n - (n-1)d$.

★ Arithmetic means divide an interval into equal parts — think of them like equally spaced points on a number line.